CHAPTER 7

7 Computer-Generated Holograms and Optical Information Processing

Abstract: A remarkable property of lenses and parabolic mirrors is their ability of performing, in parallel and at the speed of light, Fourier transform of input wave fronts and to act as “chirp”- spatial light modulator. This basic property enables creating optical information processing systems for implementation of optical Fourier analysis, image convolution and correlation. In this chapter, basic properties of lenses and parabolic mirrors relevant to optical information processing are briefly explained, elements of the theory of optical correlators for reliable target location in images are an introduced and their different implementations of optical correlators are described.

7.1 Principles of optical information processing

7.1.1 Lens as a spatial light modulator

Consider schematic diagram presented in Figs. 7.1.

Fig. 7.1. Lens as a “chirp” spatial light modulator that converts spherical wave front to plane one

One can see from the figure that coherent light from a point source at the lens focal point arrives at a point with coordinate $\xi$ in the lens plane with a phase shift $\pi \sqrt{F^2 + \xi^2} / \lambda$ with respect to the source phase, where $F$ is the lens focal distance and $\lambda$ is the light wave length. If the size of the lens is sufficiently small than the lens focal distance, this phase shift as a function of coordinate $\xi$ can be approximated as $\pi \xi^2 / \lambda F$. The fundamental property of lenses is that lenses convert a spherical wave front propagating from a point source located at the lens focal plane into a plane wave front. This implies that the lens in the set up of Fig. 7.1 acts as a spatial light modulator with transfer function $\exp\left( -i \pi \xi^2 / \lambda F \right)$. Exponential function of quadratic imaginary variable is called “chirp”-function, hence the name “chirp modulator” frequently used to characterize this property of lens as a spatial light modulator.
7.1.2 Lenses and parabolic mirrors as a Fourier transformers

Schematic diagram in Fig. 7.2 illustrates the property of lenses to perform integral Fourier transform. Consider a spherical wave front propagating from a point source at coordinate $x$ in the frontal focal plane of the lens. The lens converts this wave front into a plane wave front propagating to the rear focal plane of the lens with a slope $x/F$, so that at a point with coordinate $f$ at the lens rear focal plane it has a phase shift $2\pi x/\lambda F$ with respect to its phase on the optical axis at point $f = 0$. This implies that the point spread function $PSF(x, f)$ that describes wave propagation between frontal and rear focal planes of the lens is $\exp\left(\frac{2\pi}{\lambda F} fx\right)$, which is the kernel of the integral Fourier transform.

![Fig. 7.2. Lens as Fourier processor that converts amplitude distribution of the wave front in its fore focal plane into distribution of its Fourier transform in the rear focal plane](image)

Parabolic mirrors illustrated in Fig. 7.3, similarly to lenses, also convert a spherical wavefront emanating from the mirror focal point into a plane wavefront and, reciprocally, a plane incoming wavefront into a spherical one converging in the focal point.

![Fig. 7.3. Parabolic mirror as a “chirp” spatial light modulator and Fourier transformer](image)

Therefore parabolic mirrors also act with respect to incoming wavefronts of coherent light as Fourier transformers and chirp spatial light modulators.
7.1.3 Electro-optical image processing systems

Schematic diagram of a classical electro-optical image processing system that make use of described properties of lenses as Fourier transformers is presented in Fig. 7.4. Fig. 7.5 shows a schematic diagram of a system in which lenses are replaced by a parabolic mirror as a Fourier transformer. This replacement allows making the system more compact.

![Fig. 7.4. F electro-optical Fourier processor for image spatial filtering](image1)

![Fig. 7.5. Parabolic mirror electro-optical Fourier processor for image spatial filtering](image2)

The systems work as following. Input images recorded on the input spatial light modulator are illuminated by a parallel beam of laser light and Fourier transformed by the first Fourier lens in the system of Fig. 7.4 or by the left side of the parabolic mirror in the system of Fig. 7.5. Image spectrum in the system Fourier plane is modulated by its amplitude and phase by a spatial filter recorded on a spatial light modulator installed in the Fourier plane. The modified by the spatial filter image spectrum is then Fourier transformed by the second Fourier lens in the lens system or, correspondingly, by the right part of the parabolic mirror to form an output image on the output image sensor. In the parabolic mirror system, spatial light modulator
installed in the mirror focal plane has to be a reflective one. In this system, input image plane, output image plane and Fourier plane coincide, which makes the system much more compact than the lens based system.

In both systems, recording input image and spatial filter as well as reading output is controlled by a computer. As spatial filters, computer-generated holograms can be used.

Such image processing systems can, in principle, implement image convolution with any kernel specified by the spatial filter in the system Fourier plane. In particular, they can be used to compute correlation of input images with template images. This is an application that attracted most attention of researches ([1-4]). We will address it in the section that follows.

7.2 Optical correlators for target location: elements of the theory

Consider the task of target location in images. Let \( b(x, y) \) be an observed image that contains a target image \( a(x - x_0, y - y_0) \) in coordinates \( (x_0, y_0) \) (Fig. 7.6). The task is estimating target coordinates \( (x_0, y_0) \) by mean of processing the observed image.

![Fig. 7.6. Examples of template and input images for target location](image)

There are many applications, in which observed images can be modeled as a sum

\[
b(x, y) = a(x - x_0, y - y_0) + n(x, y)
\]

of a template image \( a(x - x_0, y - y_0) \) and a realization \( n(x, y) \) of additive Gaussian noise. In this **additive gaussian noise (AGN-)** model, the noise component is usually attributed to random interferences caused by the image sensor. Because of statistical nature of noise, the only option for estimating parameters \( (x_0, y_0) \) is applying the statistical approach to optimal parameter estimation [5]. The best statistical estimates
of parameters are the Maximum A-Posteriori Probability Estimation (MAP-estimates) and Maximum Likelihood (ML-estimates) Estimates. One can show ([6,7]) that, for the AGN-model, MAP and ML estimates \(\hat{x}_0, \hat{y}_0\) of target coordinates are solutions of the equations, respectively:

\[
\{(\hat{x}_0, \hat{y}_0)\} = \arg \max_{x_0, y_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b(x, y) a(x-x_0, y-y_0) \, dx \, dy + N_0 \ln P(x_0, y_0),
\]

(7.2.2)

\[
\{(\hat{x}_0, \hat{y}_0)\} = \arg \max_{x_0, y_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b(x, y) a(x-x_0, y-y_0) \, dx \, dy,
\]

(7.2.3)

where \(N_0\) is spectral density of noise and \(P(x_0, y_0)\) is probability density of target coordinates.

Thus, optimal ML-estimator should compute cross-correlation function between the target signal \(a(x-x_0, y-y_0)\) and the observed signal \(b(x, y)\) and take the coordinates of the maximum in the correlation pattern as the estimate. The optimal MAP-estimator also consists of a correlator and a decision-making device locating the maximum in the correlation pattern. The only difference between ML- and MAP-estimators is that, in the MAP-estimator, the correlation pattern is biased by the appropriately normalized logarithm of the object coordinates' a priori probability distribution.

The correlation operation for a mixture of signal and noise with a copy of the signal is often called matched filtering. Correspondingly, the filter that implements this operation is called matched filter. Schematic diagram of such an estimator is shown in Fig. 7.7.

The correlation operation can be implemented in Fourier transform frequency domain by multiplication of the input signal spectrum by frequency response of the matched filter which, according to the properties of Fourier transform, is \(\alpha^* (f_x, f_y)\), a complex conjugate to the target signal spectrum \(\alpha (f_x, f_y)\). As it was mentioned above, the correlation, or matched filter type of the localization devices can be implemented by optical and holographic means.

![Fig. 7.7. Schematic diagram of the ML optimal device for target location in images](image-url)
In most of other applications, target objects should be located in images, which contain many other objects that may obscure the target object. For such images, the AGN model is not appropriate and it very frequently happens that cross-correlation between the target and other non-target objects exceeds the target object autocorrelation of which leads in the correlational target location system shown in Fig. 7.7 to frequent false detection. This phenomenon is illustrated in Fig. 7.8.

![Suppose that the false objects do not overlap one another or the given object. The most obvious illustration of such a situation would be the task of locating automatically a specific character on a page of printed text.](image)

**Fig. 7.8.** Detection, by means of matched filtering, of a character with and without non-target characters in the area of search. Upper row, left image: noisy image of a printed text with standard deviation of additive noise 15 (within the signal range 0-255). Upper row, right image: results of detection of character “o” (right); one can see also quite a number of false detections of character “p” which are confused with character “o”. Bottom row, left: a noisy image with a single character “o” in its center; standard deviation of the additive noise is 175. Highlighted center of the right image shows that the character is correctly localized by the matched filter.

In such applications, background non-target objects represent the main obstacle for reliable target localization, not the sensor’s noise. In the rest of this section we show how one can the above correlation device optimal for the AGN model to minimize the danger of false identification of the target object with one of the non-target objects.

From a general point of view, this device is a special case of devices that consist of a linear filter followed by a unit for locating the highest peak of the signal at the filter output (Figure 7.9).

![Input image → Linear filter → A device for locating signal highest peak](image)

**Fig. 7.9.** Block diagram of a model of optical correlators for target detection and localization.
If we restrict ourselves to such devices, which have above described electro-optical implementation, the linear filter should be optimized so as to minimize, on average over unknown target coordinates and image sensor’s noise, the rate of false detections that occur in points, where signal at the filter exceeds the signal value at the point of the target location.

Let $b_0$ be the filter output signal value at the point $(x_0, y_0)$ of target location and $p(b_{bg}/(x_0, y_0))$ be the probability density of the filter output signal $b_{bg}(x, y)$ over the part of the image that does not contain the target object. Then the rate of false detections on average $AV_{Sens}AV_{TrgCoord}$ the image sensor’s noise (averaging operator $AV_{Sens}$) and over unknown target coordinates $(x_0, y_0)$ (averaging operator $AV_{TrgCoord}$) can be computed as

$$P_{fd} = AV_{Sens}AV_{TrgCoord} \left[ \int b_0 \, p(b_{bg}/(x_0, y_0)) \, db_{bg} \right] =$$

$$AV_{Sens}AV_{TrgCoord} \left[ \int_{AV_{Sens}(b_0)} \int b_{bg} \, p(b_{bg}/(x_0, y_0)) \, db_{bg} \right]$$

(7.2.4)

This rate has to be minimized by the appropriate design of the linear filter.

For target signal $a(x-x_0, y-y_0)$ located at coordinates $(x_0, y_0)$, output of the filter with frequency response $H(f_x, f_y)$ at the point $(x_0, y_0)$ of the target location can be computed as

$$b_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha(f_x, f_y)H(f_x, f_y) \, df_x \, df_y,$$

(7.2.5)

where $\alpha(f_x, f_y)$ is Fourier spectrum of the target signal $a(x,y)$.

In order to analytically design the filter that minimizes the rate of false alarms, one needs a relationship between the filter frequency response and the probability density of the filter output signal given filter input signal. However, such a relationship is not known. It is known only that, by virtue of the central limit theorem, linear filtering tends to normalize distribution function of output signal and that the explicit dependence of $p(b_{bg}/(x_0, y_0))$ on the filter frequency response $H(f_x, f_y)$ can only be written for the second moment $m_2$ of the distribution using Parseval’s relation for the Fourier transform:
\[ m^2 = \int_{-\infty}^{\infty} b_{bg}^2 p\left(b_{bg} \mid (x_0, y_0)\right) db_{bg} = \int_{-\infty}^{\infty} b_{bg}^2 \left(x, y/x_0, y_0\right) dx dy = \int \int |\beta_{bg}^{(0,0)}(f_x, f_y)|^2 \left|H(f_x, f_y)\right|^2 df_x df_y, \quad (7.2.6) \]

where \( \beta_{bg}^{(0,0)}(f_x, f_y) \) is Fourier spectrum of this background part of the image. Correspondingly, the second moment of the averaged probability distribution function
\[ AV_{Sens} AV_{TrgCoord} \left[ p\left(b_{bg} \mid (x_0, y_0)\right)\right] \]
can be computed as
\[ AV_{Sens} AV_{TrgCoord} \left[ m^2 \right] = \int_{-\infty}^{\infty} b_{bg}^2 AV_{Sens} AV_{TrgCoord} \left[h\left(b_{bg} \mid (x_0, y_0)\right)\right] db_{bg} = AV_{Sens} AV_{TrgCoord} \left[ \int_{-\infty}^{\infty} b_{bg}^2 \left(x, y/x_0, y_0\right) dx dy \right] = AV_{Sens} AV_{TrgCoord} \left[ \int \int |\beta_{bg}^{(0,0)}(f_x, f_y)|^2 \left|H(f_x, f_y)\right|^2 df_x df_y \right], \quad (7.2.7) \]

Therefore, for the analytical optimization of the localization device filter we will rely upon the Tchebyshev's inequality
\[ \textbf{Probability} \left(x \geq \bar{x} + b_0\right) \leq \sigma_x^2 / b_0^2. \quad (7.2.8) \]

that connects the probability that a random variable \( x \) exceeds some threshold \( b_0 \) and the variable's mean value \( \bar{x} \) and standard deviation \( \sigma_x \).

Applying this relationship to Eq.(7.2.4), we obtain:
\[ \bar{P}_u = \int_{-\infty}^{\infty} AV_{Sens} AV_{TrgCoord} \left[p\left(b_{bg}\right)\right] db_{bg} \leq AV_{bg} \frac{AV_{Sens} AV_{TrgCoord} \left[m^2 - \bar{b}^2\right]}{\left(\int AV_{Sens} \left[\bar{b}_0 - \bar{b}\right]^2\right)}, \quad (7.2.9) \]

where \( \bar{b} \) is mean value of the distribution function \( p\left(b_{bg} \mid (x_0, y_0)\right)\).

By virtue of the properties of the Fourier Transform, the mean value of the background component of the image, \( \bar{b} \), is determined by the filter frequency response \( H(0,0) \) at the point \( (f_x = 0, f_y = 0) \). Its value defines a constant bias of the signal at the filter output, which is irrelevant for the device that localizes the signal maximum. Therefore, one can choose \( H(0,0) = 0 \) and disregard \( \bar{b} \) in Eq. (7.2.9). Then we can conclude that, in order to minimize the rate \( \bar{P}_u \) of false detection errors, a filter should be found that maximizes the ratio of its response \( \bar{b}_0 \) to the target object to standard deviation \( \left(m^2\right)^{1/2} \) of its response to the background image component.
We will refer to this ratio as signal-to-clutter ratio (SCIR).

From the Schwarz’s inequality it follows that

$$
\int \int \alpha(f_x, f_y) H(f_x, f_y) df_x df_y = \int \int \frac{AV_{\text{sens}}[\alpha(f_x, f_y)]}{\left(\frac{1}{2} AV[\alpha_{bg}^0(f_x, f_y)]^2\right)^{1/2}} \left(\frac{AV[\alpha_{bg}^0(f_x, f_y)]^2}{\left(\frac{1}{2} AV[\alpha_{bg}^0(f_x, f_y)]^2\right)^{1/2}}\right)^{1/2} H(f_x, f_y) df_x df_y \leq \left\{ \int \int \left[\frac{AV_{\text{sens}}[\alpha(f_x, f_y)]}{AV[\alpha_{bg}^0(f_x, f_y)]^2}\right]^2 df_x df_y \right\}^{1/2} \left\{ \int \int H(f_x, f_y)^2 \left(\frac{AV[\alpha_{bg}^0(f_x, f_y)]^2}{\left(\frac{1}{2} AV[\alpha_{bg}^0(f_x, f_y)]^2\right)^{1/2}}\right)^{1/2} df_x df_y \right\}^{1/2}.
$$

(7.2.11)

with equality taking place when

$$
H(f_x, f_y) = \frac{AV_{\text{sens}}[\alpha^*(f_x, f_y)]}{AV[\alpha_{bg}^0(f_x, f_y)]^2},
$$

(7.2.12)

where asterisk * denotes complex conjugation and $AV[\cdot]$ denotes $AV_{\text{sens}}AV_{\text{TrgCoord}}[\cdot]$. Therefore signal-to-clutter ratio is

$$
\text{SCIR} \leq \left\{ \int \int \frac{AV_{\text{sens}}[\alpha(f_x, f_y)]^2}{AV_{\text{sens}} AV_{\text{bg}}[\alpha_{bg}^0(f_x, f_y)]^2} df_x df_y \right\}^{1/2}
$$

(7.2.13)

reaching its maximum for the optimal filter defined by the equation:

$$
H_{opt}(f_x, f_y) = \frac{AV_{\text{sens}}[\alpha^*(f_x, f_y)]}{AV_{\text{sens}}AV_{\text{TrgCoord}}[\alpha_{bg}^0(f_x, f_y)]^2}.
$$

(7.2.14)
This filter will be optimal for the particular observed image. Because its frequency response depends on the image background component the filter is adaptive. We will call this filter "optimal adaptive correlator".

To be implemented, the optimal adaptive correlator needs knowledge of power spectrum of the background component of the image. However coordinates of the target object are not known. Before the target object is located one cannot separate it from the background image component and, therefore, can not exactly determine the background component power spectrum and implement the exact optimal adaptive correlator. Yet, one can attempt to approximate the latter by means of an appropriate estimation of the background component power spectrum from the observed image.

There might be different approaches to substantiating spectrum estimation methods using additive and implant models for representation of the target and background objects within images.

In the additive model, input image \( b(x, y) \) is regarded as an additive mixture of the target object \( a(x - x_0, y - y_0) \) and image background component \( a_{bg}(x, y) \):

\[
b(x, y) = a(x - x_0, y - y_0) + a_{bg}(x, y), \quad (7.2.15)
\]

where \((x_0, y_0)\) are unknown coordinates of the target object. Therefore power spectrum of the image background component averaged over the set of possible target locations can be estimated as

\[
AV_{TrgCoord} \left[ \alpha_{bg}(f_x, f_y) \right]^2 = |\beta(f_x, f_y)|^2 + |\alpha(f_x, f_y)|^2 + \\
\beta(f_x, f_y) \alpha^{*}(f_x, f_y) AV_{TrgCoord} \left[ \exp[i2\pi(f_x x_0 + f_y y_0)] \right] + \\
\beta^{*}(f_x, f_y) \alpha(f_x, f_y) AV_{TrgCoord} \left[ \exp[-i2\pi(f_x x_0 + f_y y_0)] \right], \quad (7.2.16)
\]

Functions \( AV_{TrgCoord} \left\{ \exp[i2\pi(f_x x_0 + f_y y_0)] \right\} \) and \( AV_{TrgCoord} \left\{ \exp[-i2\pi(f_x x_0 + f_y y_0)] \right\} \) are Fourier transforms of the distribution density \( p(x_0, y_0) \) of the target coordinates:

\[
AV_{TrgCoord} \left\{ \exp[\pm i2\pi(f_x x_0 + f_y y_0)] \right\} = \int_{x} \int_{y} p(x_0, y_0) \exp[\pm i2\pi(f_x x_0 + f_y y_0)] dx_0 dy_0 \quad (7.2.17)
\]

In the assumption that the target object coordinates are uniformly distributed within the input image area, these functions are sharp peak functions with maximum at \( f_x = f_y = 0 \) and negligible values for all other frequencies. We agreed above that
point \( f_x = f_y = 0 \) is not relevant for the filter design because filter frequency response in this point defines filter output signal constant bias. Therefore, for the additive model of the target object and image background component, averaged power spectrum of the background component may be estimated as:

\[
AV_{\text{TrgCoor}} \left[ \alpha_{bg}^{(0,0)} \left( f_x, f_y \right) \right]^2 \equiv \left| \beta \left( f_x, f_y \right) \right|^2 + \left| \alpha \left( f_x, f_y \right) \right|^2; \quad f_x, f_y \neq 0. \tag{7.2.18}
\]

for all relevant points \( f_x, f_y \neq 0 \) in frequency domain.

The implant model assumes that

\[
a_{bg} (x, y) = b(x, y) w(x - x_0, y - y_0), \tag{7.2.19}
\]

where \( w(x - x_0, y - y_0) \) is a target object outlining window function:

\[
w(x, y) = \begin{cases} 0 & \text{within the target object} \\ 1 & \text{elsewhere} \end{cases}. \tag{7.2.20}
\]

In a similar way as it was done for the additive model and in the same assumption of uniform distribution of target coordinates over the image area, one can show that in this case power spectrum of the background image component averaged over all possible target coordinates can be estimated as

\[
AV_{\text{TrgCoor}} \left[ \alpha_{bg}^{(0,0)} \left( f_x, f_y \right) \right]^2 \equiv \int \int \left| W \left( p_x, p_y \right) \right|^2 \left| \beta \left( f_x - p_x, f_y - p_y \right) \right|^2 \, dp_x \, dp_y, \tag{7.2.21}
\]

where \( W \left( f_x, f_y \right) \) is Fourier transform of the window function \( w(x, y) \). Noteworthy that this method of estimating background component power spectrum resembles the traditional methods of signal spectra estimation that assume convolving power spectrum of the signal with a certain smoothing spectral window function \( \bar{W} \left( p_x, p_y \right) \) (see, for example, [8]).

Both models imply that, as a zero order approximation, the row input image power spectrum can be used as an estimate of the background component power spectrum:

\[
AV_{\text{TrgCoor}} \left[ \alpha_{bg}^{(0,0)} \left( f_x, f_y \right) \right]^2 \equiv \left| \beta \left( f_x, f_y \right) \right|^2. \tag{7.2.22}
\]

This approximation is justified by the natural assumption that the target object size is substantially smaller than the size of the input image and that its contribution to the entire image power spectrum on most frequencies is negligibly small with respect to that of the background image component.
As for the averaging over image sensor’s noise, one can show that, in the assumption of additive signal-independent zero mean sensor noise, this averaging will result in adding to the above estimates (7.2.18), (7.2.21) and (7.2.22) variance $\sigma_n^2$ of the noise. The same averaging over the image sensor’s noise required, according to Eq. 7.2.14, for the target spectrum does not change it under the above assumption of additive signal-independent zero mean noise. In this way we arrive at the following three modifications of optimal adaptive correlators:

$$H_{opt}(f_x, f_y) = \frac{\alpha^*(f_x, f_y)}{\beta(f_x, f_y)^2 + \alpha(f_x, f_y)^2 + \sigma_n^2}$$  \hspace{1cm} (7.2.23)

$$H_{opt}(f_x, f_y) = \frac{\alpha^*(f_x, f_y)}{\int \int W(p_x, p_y)^2 \beta(f_x - p_x, f_y - p_y)^2 dp_x dp_y + \sigma_n^2}$$  \hspace{1cm} (7.2.24)

$$H_{opt}(f_x, f_y) = \frac{\alpha^*(f_x, f_y)}{\beta(f_x, f_y)^2 + \sigma_n^2}$$  \hspace{1cm} (7.2.25)

7.3 The variety of optical correlators and comparison

Since invention of the optical correlator-matched filter by VanderLugt ([1]), a variety of optical correlators have been suggested and studied:

- **Matched filter (MF) correlator** ([1])

  $$OUTPUT = FT\left\{ FT\left( INPUT \right) \cdot FT\left( TRTobj \right)^* \right\}$$  \hspace{1cm} (7.3.1)

- **Phase-only filter (POF) correlator** ([9])

  $$OUTPUT = FT\left\{ FT\left( INPUT \right) \cdot \frac{FT\left( TRTobj \right)^*}{FT\left( TRTobj \right)} \right\}$$  \hspace{1cm} (7.3.2)

- **Phase-only (PO) correlator** ([10])

  $$OUTPUT = FT\left\{ FT\left( INPUT \right) \cdot \frac{FT\left( TRTobj \right)^*}{FT\left( INPUT \right) \cdot FT\left( TRTobj \right)} \right\}$$  \hspace{1cm} (7.3.3)

- **Optimal adaptive correlator** (OAC) ([6])
\[
\text{OUTPUT} = \mathbf{FT}\left\{ \frac{\mathbf{FT}(\text{INPUT}) \cdot (\mathbf{FT}(\text{TRTobj}))^*}{\left| \mathbf{FT}(\text{INPUT}) \right|^2} \right\} = \\
\mathbf{FT}\left\{ \frac{\mathbf{FT}(\text{INPUT})}{\left( \left| \mathbf{FT}(\text{INPUT}) \right|^2 \right)^{1/2}} \cdot \frac{(\mathbf{FT}(\text{TRTobj}))^*}{\left( \left| \mathbf{FT}(\text{INPUT}) \right|^2 \right)^{1/2}} \right\}
\]

\[\text{eq:7.3.4}\]

- **(-k)-th law nonlinear correlator ([11])**

\[
\text{OUTPUT} = \mathbf{FT}\left\{ \frac{\mathbf{FT}(\text{INPUT}) \cdot (\mathbf{FT}(\text{TRTobj}))^*}{\left( \left| \mathbf{FT}(\text{INPUT}) \right|^2 \right)^k} \right\}
\]

\[\text{eq:7.3.5}\]

- **Joint Transform Correlator ([12])**

\[
\text{OUTPUT} = \mathbf{FT}\left\{ \left| \mathbf{FT}(\text{INPUT} + \text{TRTobj}) \right|^2 \right\}
\]

\[\text{eq:7.3.6}\]

- Binarized versions of the correlators: amplitude or phase components of the correlator filter or both are binarized

In these formulas \(\mathbf{FT}(\cdot)\) denotes Fourier Transform operator, \(\text{INPUT}\), \(\text{OUTPUT}\) and \(\text{TRTobj}\) denote input image, output image and reference object image, correspondingly.

Matched filter correlators, phase-only filter correlators and phase-only correlators can be optically implemented in optical setups shown in Figs. 7.4 and 7.5 with spatial filters recorded on spatial light modulators in Fourier planes of these setups. As these spatial filters, computer generated holograms can be used.

In the case of the phase-only filter correlator, only the phase component of the matched filter is recorded. In the case of the phase-only correlator, also only the phase component of the matched filter is recorded but this filter is used to modulate spectrum of the input image, in which amplitude component is forcibly set to constant. For this, especial arrangements are required, which we will not discuss here. The reader may want to refer to Ref. 10.

Optical implementation of the optimal adaptive correlators requires using a non-linear signal transformation in the Fourier plane of the correlators. One of the option is inserting in Fourier planes of setups of Figs. 7.4 and 5 a nonlinear medium with a transfer function.
Output Amplitude = \((\text{Input amplitude})^k\) \hspace{1cm} (7.3.7)

We will refer to this type of nonlinearity as \((-k)\text{-th law nonlinearity}\. A plate with this medium can be placed at Fourier planes of the systems slightly out of focus in order to image spectrum smoothing before its non-linear transformation as it is recommended by Eq. 7.2.21 for better estimation of the background image component power spectrum. Block diagrams of the nonlinear optical correlator with \((-k)\text{-th law nonlinearity} built on the base of the lens system of Fig. 7.4 is shown in Figure 7.10. As the intensity of light incoming to the nonlinear medium is proportional to the intensity of input image power spectrum \(\beta(f_x, f_y)^2\) in Eq. 7.2.22, optimal value for the parameter \(k\) of the nonlinearity is \(k = 1\). Simulation experiments reported in Ref. 11 show that slight deviations from this optimal value are tolerable.

![Fig. 7.10. Lens based nonlinear electro-optical correlator. \(F\) – focal distance of Fourier lenses.](image)

Similar modification of the parabolic mirror system is also possible. In this case, requirements to the dynamic range of the nonlinear medium are eased because light propagates through the nonlinear medium twice, before coming to the reflective filter and after the reflection.

Described modifications of optical correlators differ in their capability of discriminating target objects from non-target objects and clutter background in images. In Figure 7.11, results of experimental comparison of the discrimination capability of above described optical correlators reported in Ref. 11 are presented. In the experiments, carried out using 16 test air photographs of 128x128 pixels presented in Fig. 7.12 and a test target image in a form of a circle of 5 pixels in diameter embedded into test images, signal-to-clutter ratio was measured for each of the correlator. In addition to all described correlators, the ideal optimal adaptive correlator was tested as well, built for exact background component of images, which was known in the simulation experiments.

Results presented in Fig. 7.9 show that optimal adaptive correlator considerably outperforms other types of correlators in terms of signal-to-clutter ratio they provide.
though still there is a two to five fold gap between its performance and that of the exact optimal adaptive correlator. This gap motivates search for better methods of estimating background component of images.

Fig. 7.11. Comparison, in terms of the signal-to-clutter ratio (SCIR), of the discrimination capabilities of the exact Optimal Adaptive Correlator (exact opt corr.); optimal adaptive correlator (nlin. opt. corr.) with background image component power spectrum estimation by means of blurring observed image power spectrum according Eq. 7.2.21; phase-only correlator (POCorr), phase-only filter correlator (POF corr), and matched filter correlator (MF corr) for a set of 16 test images of 128x128 pixels and a circular target object of 5 pixels in diameter embedded in images.

Fig. 7.12. Set of 16 test images of 128x128 pixels
7.4 Joint Transform Correlator

Lens and parabolic mirror correlators require pre-fabricated filter placed in their Fourier plane or a computer-generated hologram synthesized and recorded by computer. Joint Transform correlator is an electro-optical correlator that does not require a pre-fabricated filter. Schematic diagram of the Joint Transform Correlator (JTC) is presented on Fig. 7.13. JTC works as following. Let, as before, \( b(x,y) \) be an input image and \( a(x,y) \) be a template image that should be located in the input image. In the Joint Transform Correlator, the template image is placed aside to the input image in the same front focal plane of the first Fourier lens. The lens performs Fourier transform of the sum \( b(x,y)+a(x,y) \) of these images and forms in its rear focal plane their joint spectrum \( \gamma(f_x,f_y) = \beta(f_x,f_y) + \alpha(f_x,f_y) \). A photosensitive sensor installed in the lens rear focal plane generates an electrical signal proportional to a certain, in general nonlinear, function \( g(\gamma(f_x,f_y))^2 \) of the joint spectrum intensity:

\[
g(\gamma(f_x,f_y))^2 = g(\beta(f_x,f_y) + \alpha(f_x,f_y))^2 = \left(\beta(f_x,f_y) + \alpha(f_x,f_y)\right)^2 + \beta^2(f_x,f_y)\alpha(f_x,f_y) + \beta^2(f_x,f_y)\alpha^2(f_x,f_y) \tag{7.4.1}
\]

Fig. 7.13. Joint Transform correlator. \( F \) – focal distance of the Fourier lenses.
As one can see in Eq. 7.3.8, the last term in the argument of the function \( g(\cdot) \) is the product of the input image spectrum \( \beta(f_x, f_y) \) and complex conjugate \( \alpha^*(f_x, f_y) \) to the template image spectrum, which corresponds to matched filtering in transform domain.

Signal \( g\left[\mathcal{F}\left(f_x, f_y\right)^2\right] \) can be, in principle, further modified by the computer or directly used to record on the second SLM installed in frontal Fourier plane of the second Fourier lens of the JTC. This performs its Fourier transform and forms a correlation image in its rear Fourier plane, where it is sensed by another photosensitive sensor and put in the computer for further processing and analysis.

Input image and template image can, in principle, be optically recorded on the input SLM. Recording them and, especially, the template image, from computer is more advantageous, as it enables rapid change of templates when detection of different modifications of target images, such as scaled and rotated, is required.

If the function \( g(\cdot) \) is a linear function, as it is commonly assumed, Joint Transform Correlator can be used as a matched filter correlator. However, with an appropriate selection of the nonlinear function \( g(\cdot) \), Joint Transform Correlator can approximate the optimal adaptive correlator.

Consider frequency response (Eq. 7.2.23) of the optimal adaptive correlator obtained for the additive model of evaluating power spectrum of the background component of images and neglect in it the variance of the additive sensor’s noise:

\[
H_{\text{opt}}(f_x, f_y) = \frac{\alpha(f_x, f_y)}{\beta(f_x, f_y) + \alpha(f_x, f_y)},
\]

(7.4.2)

where \( \beta(f_x, f_y) \) and \( \alpha(f_x, f_y) \) are Fourier spectra of the image and the target object, correspondingly, and let us show that, with a logarithmic nonlinear transformation of the joint spectrum, the JTC approximates this filter. With the logarithmic nonlinearity, the transformed joint power spectrum \( \mathcal{F}\left(f_x, f_y\right)^2 \) at the output of this nonlinear device can be written as

\[
\log \left[ \mathcal{F}\left(f_x, f_y\right)^2 \right] = \log \left[ \beta(f_x, f_y) + \alpha(f_x, f_y) \right] = \log \left[ \beta(f_x, f_y) + \alpha(f_x, f_y) \right] + \log \left[ \beta(f_x, f_y) + \alpha(f_x, f_y) \right] =
\]

\[
\log \left[ \beta(f_x, f_y) + \alpha(f_x, f_y) \right] + \log \left[ \beta(f_x, f_y) + \alpha(f_x, f_y) \right] + \beta^*(f_x, f_y) \alpha(f_x, f_y) + \beta(f_x, f_y) \alpha^*(f_x, f_y) \right] \}
\]

(7.4.3)

Since the size of the reference object is usually much smaller than the size of the input image, one can assume that, for the majority of the spectral components,

\[
\left| \beta(f_x, f_y) \right|^2 + \left| \alpha(f_x, f_y) \right|^2 \gg \beta^*(f_x, f_y) \alpha(f_x, f_y) + \beta(f_x, f_y) \alpha^*(f_x, f_y) \}
\]

(7.4.4)
With this assumption, \( \log \left| \gamma(f_x, f_y) \right|^2 \) is approximately equal to
\[
\log JPS(f_x, f_y) \equiv \log \left\{ \left| \beta(f_x, f_y) \right|^2 + \left| \alpha(f_x, f_y) \right|^2 \right\} + \frac{\beta^*(f_x, f_y) \alpha(f_x, f_y)}{\left| \beta(f_x, f_y) \right|^2 + \left| \alpha(f_x, f_y) \right|^2} + \frac{\alpha^*(f_x, f_y)}{\left| \beta(f_x, f_y) \right|^2 + \left| \alpha(f_x, f_y) \right|^2} \beta^*(f_x, f_y)
\]
(7.4.5)

**Fig. 7.14.** Arrangement of input image and target object at the input of JTC and resulting correlation outputs of conventional linear (matched-filter) and nonlinear (logarithmic) JTCs (a); and central cross sections of the matched-filter and nonlinear JTCs outputs for the test input and target images shown in a) (b).
When a JTC configuration is utilized, the two last terms displaying the correlation function and its conjugate are readily separated from each other and from the first term that produces the zero order diffraction term (see the arrangement in Fig. 7.14). The last term of expression (7.4.5) exactly reproduces filtering described by Eq. 7.4.2. Therefore, one can conclude that the joint transform correlator with a logarithmic nonlinearity can be regarded as an approximation to the OAC and therefore promises an improved discrimination capability.

The logarithmic nonlinearity can be implemented in JTCs by computer processing of the joint spectrum or directly in the photosensitive sensor on an analog level. Optical sensors with a logarithmic sensitivity functions are now becoming commercially available.

References